刚体动力学:

1. 与平动的对应关系

$x \longrightarrow \theta$

$v \longrightarrow w$

$a \longrightarrow \alpha$ $\begin{cases} \text{大小} \lim_{t\to 0} \frac{\Delta w}{\Delta t} \\ \text{方向 右手法则} \end{cases}$

2. 角速度与线速度 : $v = \vec{\omega} \times \vec{r}$

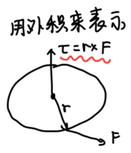
角加速度与线加速度:

$\left.\begin{aligned} a_r &= \frac{v^2}{r} = r\omega^2 \\ a_t &= r\cdot\alpha \end{aligned}\right\} \Rightarrow a = \sqrt{a_t^2 + a_r^2} = r\sqrt{\alpha^2 + w^4}$

3. 力矩 (τ) $\tau = r\cdot F\sin\phi = Fd$

- 在轴确定的情况下才能定义力矩
- 一般逆时针为正

用外积来表示



4. 转动惯量 $F_t = ma_t = m\cdot r\cdot\alpha$

$\Downarrow$

$\tau = F_t\cdot r = mr^2\cdot\alpha = I\alpha$

定义为转动惯量I

- 确定相应的轴后可以确定转动惯量

$I = \sum_i m_i r_i^2$

5. 转动动能: $K_R = \frac{1}{2}Iw^2$

转动功: $dW = F\cdot ds = F\sin\phi\cdot r d\theta = \tau d\theta$

$= I\cdot\alpha\cdot d\theta = I\cdot\frac{dw}{dt}\cdot d\theta = I\cdot w dw$

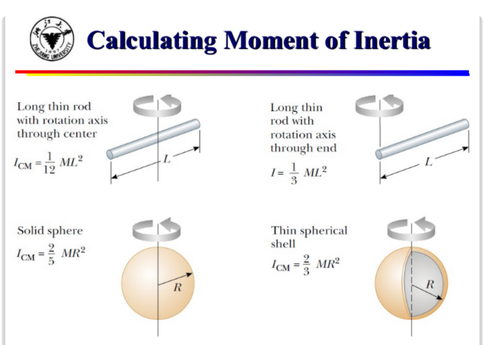
$W = \int_{\theta_i}^{\theta_f}\tau d\theta = \int_{w_i}^{w_f} Iw dw = \frac{1}{2}Iw_f^2 - \frac{1}{2}Iw_i^2$

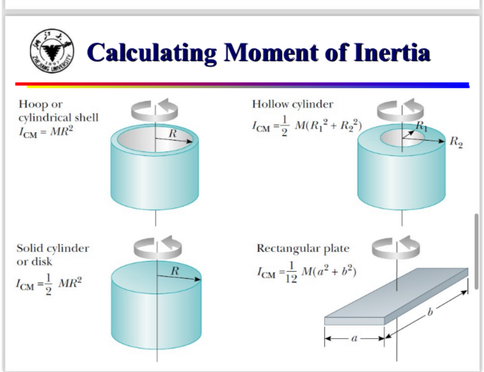
转动功率 $P = \tau w$

TABLE 10.3 Useful Equations in Rotational and Linear Motion

Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Resultant torque $\Sigma\tau = I\alpha$	Resultant force $\Sigma F = ma$
If α = constant: $\omega_f = \omega_i + \alpha t$; $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$; $\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	If a = constant: $v_f = v_i + at$; $x_f - x_i = v_i t + \frac{1}{2}at^2$; $v_f^2 = v_i^2 + 2a(x_f - x_i)$
Work $W = \int_{\theta_i}^{\theta_f}\tau d\theta$	Work $W = \int_{x_i}^{x_f} F_x dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Resultant torque $\Sigma\tau = dL/dt$	Resultant force $\Sigma F = dp/dt$

6. 常见物体的转动惯量





7. 地转偏向力 (南左北右)

$F_k = 2m\vec{\omega}\times\vec{v_k}$

$\vec{F}_{net} = \vec{F} - 2m\vec{\omega}\times\vec{v_k} + m\omega^2\vec{r}$

8. 平行轴定理 (非重心处轴转动惯量的计算)

$I = I_{cm} + MD^2$

- 滚动圆柱的总动能

$K = \frac{1}{2}Iw^2$

$= \frac{1}{2}I_{cm}w^2 + \frac{1}{2}MV_{cm}^2$

9. 角动量 $L = r\times P$ $\sum\tau_{exp} = \frac{dL}{dt}$

$= Iw$ $= I\alpha$.

在 $\sum\tau_{ext} = 0$ 时, 角动量守恒

在孤立环境下, 几个守恒式

① 机械能守恒

② 角动量守恒

③ 动量守恒

波动力学:

1. 简谐运动: $F_s = -kx = ma$

$\Rightarrow$ 解: $x = A\cos(wt + \phi)$ $w = \sqrt{k/m}$

$T = \frac{2\pi}{w}$ $f = \frac{1}{T} = \frac{w}{2\pi}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}mw^2A^2\sin^2(wt+\phi)$

$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(wt+\phi)$

$E = K + U = \frac{1}{2}kA^2$

2. 单摆与复摆

① 单摆

$-mg\sin\theta = m\frac{d^2s}{dt^2}$

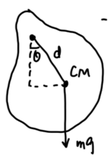
$-g\sin\theta = L\cdot\ddot{\theta}$

$\Rightarrow \ddot{\theta} = -\frac{g}{L}\theta$

$\Rightarrow \theta = \theta_{max}\cos(wt+\phi)$

$\Rightarrow T = 2\pi\sqrt{\frac{L}{g}}$

② 复摆

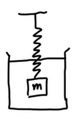


$-mgd\sin\theta = I\cdot\alpha$

$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta$

$w = \sqrt{\frac{mgd}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{mgd}}$

3. 阻尼振子



$\sum F_x = -kx - bv = ma_x$

$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$

4. 杨氏模量 $Y = \frac{F/A}{\Delta L/L_i}$

5. 连续介质中的波

(1) 波的反射

① 固定端点下.

② 自由端点下

(2) 线性波的方程 $f(x-Vt)$ (向右传播的波)

$\frac{\partial^2 u}{\partial t^2} = v^2\frac{\partial^2 u}{\partial x^2}$

① 在绳上的正弦波

$(\mu\Delta s)\cdot\frac{\partial^2 y}{\partial t^2} = T(s+\Delta s)\cdot\frac{\partial y}{\partial s}\Big|_{s+\Delta s} - T(s)\cdot\frac{\partial y}{\partial s}\Big|_s$

$= T\cdot\frac{\partial^2 y}{\partial s^2}\cdot\Delta s$

$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\cdot\frac{\partial^2 y}{\partial s^2}$

$\Rightarrow v = \sqrt{\frac{T}{\mu}}$

② 正弦波的一般式

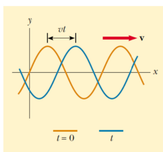
- **The linear wave equation**

$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$

- **The most important family of the solutions are**

$y = A\sin(kx - \omega t + \phi)$

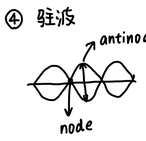
amplitude; angular wave number; angular frequency; phase constant (注意是负号)



$\omega = vk$

③ 能量传递 $P = \frac{1}{2}\mu v(wA)^2$ 线上能量传递

④ 驻波



$k\frac{\lambda}{2} = L$

⑤ 在空气中传播 $c = \sqrt{k} = \sqrt{\frac{dp}{d\rho}}$

固体中传播 $c = \sqrt{\frac{B}{\rho_0}}$

⑥ 强度 $I = \frac{1}{2}\rho v(wS_{max})^2$

6. 多普勒效应

$f' = \frac{c+V_o}{c-V_s}f$

相对论:

1. 速度的合成

$\Rightarrow w = \frac{u+v}{1+(\frac{u}{c})(\frac{v}{c})}$

$\Rightarrow \frac{c-w}{c+w} = \left(\frac{c-u}{c+u}\right)\left(\frac{c-v}{c+v}\right)$

2. 钟慢效应与尺缩效应

① $\Delta t = \gamma\Delta t'$ $\beta = v/c$ $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

② $L = L_0/\gamma$

3. 洛伦兹变换 (把原先参考系里的量换到另一个参考系中去)

$\begin{cases} x' = \frac{x - Vt}{\sqrt{1-v^2/c^2}} \\ t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-v^2/c^2}} \end{cases}$

4. Invariant interval (时空距离)

$s^2 = -(ct)^2 + x^2 + y^2 + z^2$

5. 相对论下的多普勒效应

$$\cos(\omega t - kx) = \cos\left[\omega\frac{t' - vx'/c^2}{\sqrt{1-v^2/c^2}} - k\frac{x' - vt'}{\sqrt{1-v^2/c^2}}\right]$$
$$= \cos\left[\frac{\omega + kv}{\sqrt{1-v^2/c^2}}t' - \frac{k + v\omega/c^2}{\sqrt{1-v^2/c^2}}x'\right]$$
$$= \cos[\omega' t' - k' x']$$

又 $w = ck \Rightarrow w' = w\cdot\frac{1+v/c}{\sqrt{1-v^2/c^2}} = w\sqrt{\frac{1+v/c}{1-v/c}}$

$f = \sqrt{\frac{1+v/c}{1-v/c}}f_0$

6. 相对论力学

质量调整 $m_i = \frac{m}{\sqrt{1-v^2/c^2}}$ (m为静质量)

其余公式与牛顿力学类似, $\vec{p} = m_i\vec{v}$ $\vec{F} = \frac{d}{dt}(m_i\vec{v})$

- $p = m(v)\cdot v = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$ (利用全同粒子的完全非弹性碰撞 $\begin{cases}\text{质量守恒} \\ \text{动量守恒}\end{cases}$)
- f 定义为 $\frac{dp}{dt}$
- $E_k = \frac{m_0v^2}{\sqrt{1-v^2/c^2}} - m_0c^2 = (m - m_0)c^2$

静质能: m_0c^2

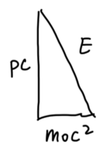
总能量: $E = E_k + m_0c^2 = mc^2 = \frac{m_0c^2}{\sqrt{1-v^2/c^2}}$ $\Rightarrow$ 质量关系

- 能量与动量的关系

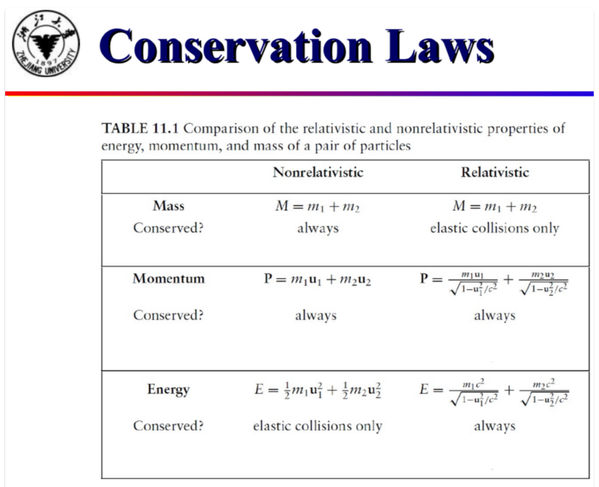
$m^2 = \frac{m_0^2}{1-v^2/c^2} \Rightarrow m^2c^4 - m^2v^2c^2 = m_0^2c^4$

$\downarrow$

$E^2 - p^2c^2 = m_0^2c^4$



没有质量的物体 $m_0 = 0$ $\Rightarrow E = pc$ 可定义四维动量 $\mathbb{P} = (E/c, p)$



热学:

1. 膨胀 $\left.\begin{aligned}\Delta L &= \alpha L_0\Delta T \\ \Delta V &= \beta V_0\Delta T\end{aligned}\right\} \Rightarrow \beta = 3\alpha$

2. 范德华方程

$(p + \frac{aN}{V^2})(V - Nb) = Nk_BT$

$Nk_B = nR$ (N:分子数 n:摩尔数)

$k_B = R/N_A$

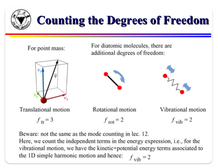
a 与 分子量/极性有关 b 与分子大小有关

理想气体适用条件: 稀疏气体, 高温

3. 理想气体的内能

$U = f\cdot\frac{Nk_BT}{2}$ f为自由度

$C_V = \left(\frac{\Delta U}{\Delta T}\right) = \frac{f}{2}Nk_B$



4. 分子速度: $V_{mp} = \sqrt{\frac{2k_BT}{m}}$ 最大值

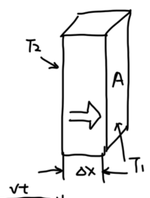
$V_{rms} = \sqrt{\frac{3k_BT}{m}}$ 方均根

$V_{ave} = \sqrt{\frac{8k_BT}{\pi m}}$ 平均值

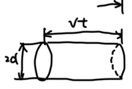
5. 热传递

$P = \frac{Q}{\Delta t} \propto A\frac{\Delta T}{\Delta x}$

$= -K_t A\frac{dT}{dx}$



6. 平均自由程



$l = \frac{Vt}{n_v\pi d^2(\sqrt{2}V)t} = \frac{1}{\sqrt{2}n_v\pi d^2}$ (在这样的路程下会遇上多个粒子 $(n_v\pi d^2\sqrt{2}vt)$)

(vt)

相对速度

7. 热力学第一定律 $dU = dQ - dW$

8. C_p 与 C_v

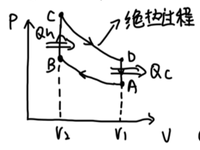
等压 $Q = \Delta U + W = C_v\Delta T + P\Delta V = C_v\Delta T + Nk_B\Delta T$

$\Rightarrow C_p = \left(\frac{Q}{\Delta T}\right)_p = C_v + Nk_B$

$\gamma = C_p/C_v = \frac{f+2}{f}$

9. 一些热机

(1) Otto Cycle $W = Q_h - Q_c$



$Q_h = nC_v(T_C - T_B)$

$Q_c = nC_v(T_D - T_A)$

$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_D - T_A}{T_C - T_B}$

理想气体绝热过程

$C_vdT = dU = -PdV$ $\gamma = \frac{C_p}{C_v} = 1 + \frac{Nk_B}{C_v}$

$\frac{C_vdT}{T} = -\frac{Nk_BdV}{V}$

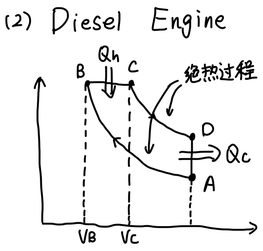
$\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0$

$\Rightarrow TV^{\gamma-1} = const$

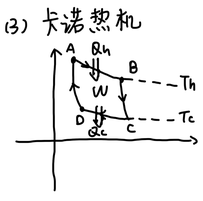
$PV^{\gamma} = const$

$\left.\begin{aligned} T_AV_1^{\gamma-1} &= T_BV_2^{\gamma-1} \\ T_DV_1^{\gamma-1} &= T_CV_2^{\gamma-1} \\ \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \frac{T_A}{T_B} = \frac{T_D}{T_C} \end{aligned}\right\} \Rightarrow e = 1 - \frac{1}{(V_1/V_2)^{\gamma-1}}$

(2) Diesel Engine



(3) 卡诺热机



$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$

$Q_h = nRT_h\ln\frac{V_B}{V_A}$

$Q_c = nRT_c\ln\frac{V_C}{V_D}$

$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}\frac{\ln(V_C/V_D)}{\ln(V_B/V_A)} = \frac{T_c}{T_h}$

卡诺热机效率仅与温度有关, 且不可能有比卡诺热机效率更高的热机

10. 熵 (状态函数)

- $dS = \frac{dQ_{reversible}}{T}$ ⇐ 注意: 这个Q是可逆的
- $dU = TdS - PdV$
- 玻尔兹曼定义 $S = k_B\ln W$ ← 微观状态数

	等体积	等压	等温	绝热	绝热自由
ΔU	$nC_v\Delta T$	$nC_v\Delta T$	0	$-W$	0
ΔW	0	$P\Delta V$	$nRT\ln V_1/V_2$	W	0
ΔQ	$nC_v\Delta T$	$P\Delta V + nC_v\Delta T$	$nRT\ln V_1/V_2$	0	0
ΔS	$nC_v\ln T_1/T_2$	$nC_p\ln T_1/T_2$	$nR\ln V_1/V_2$		$nR\ln V_1/V_2$ (多同绝热过程)